International Journal of Theoretical Physics, Vol. 42, No. 6, June 2003 (© 2003)

Quantum Stabilization of Moduli in a Slice of AdS_6 Compactified on S^1

Oriol Pujolàs[1]

Received April 14, 2003

We review the effective potential due to massive bulk scalar fields in higher-dimensional warped brane models found in Flachi *et al.* (Quantum stabilization of moduli in higher dimensional brane models, arXiv:hep-th/0301???, 2003) specializing it to a slice of AdS_6 compactified on the circle. This model contains two moduli that parametrize the interbrane distance and the size of S^1, or equivalently the positions of the two branes. Their values determine the Planck/EW hierarchy, in a combination of large volume and redshift effects. It is found that the observed hierarchy is compatible with both moduli stabilized by the Casimir forces without fine-tuning (except for the one needed to match the cosmological constant). This contrasts with the Randall–Sundrum model, where gauge fields in the bulk are needed.

KEY WORDS: extra dimensions; brane models; hierarchy problem; moduli stabilization.

1. INTRODUCTION

The brane world scenario (Antoniadis *et al.*, 1998; Arkani-Hamed *et al.*, 1998, 1999; Randall and Sundrum, 1999) has generated a great interest in the last years, mainly thanks to its phenomenological applications in both particle physics and cosmology. In this paper, we concentrate our attention on the understanding of hierarchy problem that these models offer. Arkani-Hamed *et al.* (1998, 1999) realized that in models with n extra dimensions of typical size L, fundamental cutoff given by M, and matter confined on a $3 + 1$ brane, the effective four-dimensional (4D) Planck mass is given by $m_P^2 \sim (LM)^n M^2$. Taking the cutoff $M \sim$ TeV, and the present upper bound for $L \sim \mu m$, we can easily obtain the large value for $m_P \sim 10^{16}$ TeV. Since in this mechanism the size of the bulk space is large compared to the fundamental scale $1/M$, we shall call this a *large volume* effect.

The Randall–Sundrum (RS) model (Randall and Sundrum, 1999) proposes another very interesting possibility. In this case, there are two branes placed at the

[1] IFAE, Departament de Física, Universitat Autònoma de Barcelona, 08193 Bellaterra (Barcelona), Spain; e-mail: pujolas@ifae.es.

0020-7748/03/0600-1369/0

ends of a warped space forming a slice of AdS_5. All the mass scales are of order m_{P}, and matter is assumed to live on the negative tension brane (with the smaller wrap factor). In the 4D effective theory, the masses of the fields are redshifted by the ratio of wrap factors $a = e^{-d/\ell}$, where d is the proper interbrane distance, and ℓ is the curvature radius of AdS_5.

As in any model with extra dimensions, a set of scalar fields parametrizing the background configuration (the *moduli*) appear in the effective 4D theory. At the classical level, they are massless and some stabilization mechanism is required in order for the whole setup to be consistent.

In the RS model, there is only one modulus (the *radion*) parametrizing at the same time the interbrane distance and the electroweak/Planck hierarchy. Goldberger and Wise (1999) proposed a stabilization mechanism essentially consisting in a bulk classical scalar field with an ad hoc potential. This can generate a sizeable mass for the radion compatible with a large hierarchy with one fine-tuning corresponding to the cosmological constant. The possibility that the Casimir energy can stabilize naturally the radion was studied in the literature (Flachi and Toms, 2001; Garriga *et al.*, 2000; Goldberger and Rothstein, 2000; Toms, 2000). It was found that for gravitons and generic scalar fields, a fine-tuning is needed when a large hierarchy is present (besides the tuning corresponding to the cosmological constant), and moreover the mass for the radion is very small. In a recent work (Garriga and Pomarol, 2002), it has been shown that the contribution due to bulk gauge fields can stabilize the radion without fine-tuning.

The presence of warp factors in brane world models is a generic feature of theories related to M theory. Some of these can be described by supergravity theories with bulk scalar fields, and contain vacuum solutions (see, e.g., Brax *et al.*, 2002; Yaum, 2000, 2001) with warp factors different to the exponential present RS. In particular, five-dimensional (5D) models with a power law warp factor were considered in Garriga *et al.* (2001). The computation and renormalization of the effective potential is somewhat more involved in this case. It was found that for sufficiently steep warp factor, the two moduli present could be stabilized without fine-tuning. It was realized in Garriga *et al.* (2001) that some models with more extra dimensions reduce to this kind of theories once reduced to five dimensions.

Then, it is interesting to consider more general configurations with more extra dimensions or even with branes of codimension greater than one. As a first step, here we consider a simple generalization of the RS model, with a six-dimensional (6D) bulk and 5D branes so that the full configuration is a slice of AdS_6 compactified on a circle, with bulk topology $(S^1/Z_2) \times M^4 \times S^1$ and brane topology $M^4 \times S^1$. The model we are considering has the same warp factor for the internal space S^1 and the Minkowski factor, M^4. However, models with different warp factors can be found in the literature (Gregory, 2000; Randjbar *et al.*, 2000a), and will be the subject of future research (Flachi *et al.*, manuscrip in preparation). In passing, we note that the phenomenology of such a scenario has been recently considered in

Davoudiasl *et al.*, (2002) and also that these types of solution allow to naturally localize chiral fermions (Randjbar *et al.*, 2000b).

This model qualitatively mimics the configuration present in the Hořava–Witten theory (Hořava and Witten, 1996a,b; Lukas *et al.*, 1999). The vacuum configuration of this theory consists of an 11-dimensional space with topology $(S^1/Z_2) \times \mathrm{CY}_6 \times M^4$ and different warp factors for the Minkowski M^4 and the Calabi–Yau (CY_6) factors. The two branes are of codimension 1 as well. In this example the authors argued that there is a regime in which the size of the CY manifold is much smaller than the length of the orbifold, and correspondingly there is a cosmological period in which the space-time is effectively 5D. More generally, if brane world scenarios are to be relevant at all for particle physics and also link low energy physics with any more fundamental theory formulated in higher dimensions, then it is interesting to consider brane world scenarios with more "internal" space.

This paper is organized as follows. In Section 2 we present the 6D model, its compactification on the circle, and the reduction to four dimensions, using the results of Garriga *et al.* (2001). We derive the couplings of the matter located on the negative tension brane in Section 2.3. The possible values of the scales involved in the model are discussed in Section 2.4. The computation of the potential is outlined in Section 3 and given in more detail in the Appendices. We discuss the stabilization mechanism offered by this potential in Section 4. We end with the conclusions and some remarks related to Garriga *et al.* (2001) in Section 5.

Our notation is the following. We split the coordinates into the usual 4D x^μ, the coordinate along the orbifold x^5, and along the circle θ. The higher-dimensional *brane* indices $a, b \ldots$ run over μ and θ, and the 5D bulk indices (after compactification on S^1) $\alpha, \beta \ldots$ run over μ and x^5. This report is based on a work done in collaboration with Flachi *et al.* (2003).

2. THE MODEL

In this paper we consider the 6D generalization of the RS model compactified on S^1, with two codimension-1 branes. The 6D action is given by

$$S_6 = -M^4 \int d^6x\sqrt{-g}(\mathcal{R} + \Lambda) - \sigma_+ \int d^5x\sqrt{-g_+} - \sigma_- \int d^5x\sqrt{-g_-}, \quad (2.1)$$

where $\mathcal{R}$ is the curvature scalar, $g^{\pm}_{ab}$ are the induced metrics on the branes, and M is the 6D Planck mass.

The metric describing AdS$_6$ in conformal coordinate,

$$ds^2 = a^2(z)[\eta_{\mu\nu}\, dx^\mu\, dx^\nu + R^2\, d\theta^2 + dz^2] \quad \text{with} \quad a(z) = \ell/z, \quad (2.2)$$

is a solution of (2.1) provided (Flachi and Toms, 2001)

$$\Lambda = -20/\ell^2,$$
$$\sigma_\pm = \pm 16 M^4/\ell, \tag{2.3}$$

where ℓ is the curvature radius of the bulk AdS_6, and R is a fixed length scale parametrizing the size of the extra circle.

2.1. Reduction to 5D

This model reduces to that of Garriga *et al.* (2001) with $q = 4$ upon compactification on the circle, keeping its size as a dynamical scalar field. To see this, consider the following ansatz for the metric including the 5D graviton $g_{\alpha\beta}(x^\mu, y)$ and the *dilaton* $\sigma(x^\mu, y)$ (for simplicity we freeze the graviphoton corresponding to the components (μ, θ), of the metric),

$$ds^2 = g^{(s)}_{\alpha\beta}(x^\mu, y)\, dx^\alpha\, dx^\beta + e^{2\sigma(x^\mu, y)} R^2\, d\theta^2. \tag{2.4}$$

If we insert this ansatz into the action (2.1) and integrate out the θ dependence, we obtain the 5D action[2]

$$S_5 = -2\pi R \left[M^4 \int d^5x \sqrt{-g_{(s)}}\, e^\sigma \left(\mathcal{R}^{(s)} + \Lambda\right) + \sigma_+ \int d^4x \sqrt{-g_{(s)+}}\, e^\sigma \right.$$
$$\left. + \sigma_- \int d^4x \sqrt{-g_{(s)-}} e^\sigma \right]. \tag{2.5}$$

Expressed in the 5D Einstein frame, given by

$$g^{(5)}_{\alpha\beta} = e^{2\sigma/3} g^{(s)}_{\alpha\beta}, \tag{2.6}$$

and in terms of the canonical scalar field $\phi = -2\sqrt{2/3}\sigma$, we can rewrite this action as

$$S_5 = -M_5^3 \int d^5x \sqrt{-g_{(5)}} \left(\mathcal{R}^{(5)} + \frac{1}{2}(\partial\phi)^2 + \Lambda\, e^{\phi/\sqrt{6}} \right)$$
$$-\sigma_{5+} \int d^4x \sqrt{-g_{(5)+}}\, e^{\phi/2\sqrt{6}} - \sigma_{5-} \int d^4x \sqrt{-g_{(5)-}}\, e^{\phi/2\sqrt{6}}. \tag{2.7}$$

Here, $g^{(5)\pm}_{\mu\nu}$ is the metric induced by $g^{(5)}_{\alpha\beta}$, $M_5^3 = 2\pi R M^4$ is the 5D Planck mass and $\sigma_{5\pm} = 2\pi R \sigma_\pm$, the effective 5D brane tensions. This corresponds to one of the scalar tensor models considered in Garriga *et al.* (2001), with $c = -1/\sqrt{6}$. This model has a vacuum solution of the form

$$ds^2_{(5)} = a^2_{(5)}(z)(dz^2 + \eta_{\mu\nu}\, dx^\mu\, dx^\nu),$$

[2] The label$^{(s)}$ in the 5D metric signals that in this frame, the scaling symmetry present in this theory (see Garriga *et al.*, 2001) corresponds to $\sigma \to \sigma +$ const, and a scaling invariant metric $g^{(s)} \to g^{(s)}$.

$$\phi_0(z) = -\sqrt{6(\beta+1)/\beta}\,\ln a_{(5)}(z) \quad \text{with} \quad a_{(5)}(z) = (z/\ell)^\beta, \tag{2.8}$$

and a S^1/Z_2 orbifold topology for the extra dimension, the power of the warp factor $a_{(5)}(z)$ being $\beta = -4/3$. The scalar field ϕ, parametrizing the size of the internal space S^1, changes along the orbifold. From the point of view of this 5D effective theory, the warp factor is a no longer an exponential of the proper distance normal to the branes, so the bulk space is not AdS_5. The reason for this is that we have expressed the solution in the 5D Einstein frame which is related to the 6D Einstein frame through (2.6).[3]

2.2. Reduction to 4D

This solution contains two physically meaningful parameters not determined by the equations of motion, the radius R of S^1, and the proper interbrane distance $d = y_+ - y_-$. To be precise, the physical size of the internal space S^1 at the branes is given by $R_\pm \equiv a_\pm R$, which $a_\pm = a(z_\pm)$, rather than R. This suggests that we can also describe completely the two-brane system specifying $R_\pm$, or even $z_\pm$ instead.[4] This choice is particularly suitable because in terms of $R_\pm$ the effective potential has an especially simple form.

In this paper, we treat the degrees of freedom associated to these (classically) free parameters of the model in the so-called *moduli approximation*. This consists in promoting these integration constants to 4D fields or moduli. The idea is that since they correspond to flat directions in action (2.1), they can be easily excited and are relevant at low energies. To obtain the 4D effective action describing the moduli, we can proceed as in Garriga *et al.* (2001) beginning from the 5D reduced action (2.7), and promoting the brane locations $z_\pm$ to x^μ-dependent fields.[5] In terms of a 5D metric ansatz that includes the 4D graviton zero mode,

$$ds^2 = a_{(5)}^2(z)(dz^2 + \tilde{g}_{\mu\nu}(x)\,dx^\mu\,dx^\nu), \tag{2.9}$$

we can read off the result from Garriga *et al.* (2001),

$$S_4[\varphi_\pm] = -m_{\mathrm{P}}^2 \int d^4x\sqrt{-\tilde{g}}\left\{\left(\varphi_+^2 - \varphi_-^2\right)\tilde{\mathcal{R}} - \frac{16}{3}[(\tilde{\partial}_{\varphi+})^2 - (\tilde{\partial}_{\varphi-})^2]\right\}, \tag{2.10}$$

where we have introduced

$$\varphi_\pm \equiv \left(\frac{z_\pm}{\ell}\right)^{-3/2} = a_\pm^{3/2},$$

[3] Note that there are two notions of *proper* coordinate along the bulk space, the 6D and the 5D. This is not the case for the conformal coordinate.

[4] One important difference between the RS model and the model considered here is that, due to the compactified S^1 factor, the bulk is *not* homogeneous in the orbifold direction.

[5] In Flachi *et al.* (2003) it has been shown that the moduli action can be equivalently derived reducing the theory directly from six to four dimensions.

and the 4D Planck mass is given by

$$m_{\mathrm{P}}^2 = \frac{2}{3}\ell M_5^3 = \frac{4\pi}{3}\ell R M^4. \tag{2.11}$$

The modulus corresponding to the positive tension brane has a kinetic term with the "wrong" sign. As already pointed in Garriga *et al.* (2001) this does not necessarily signal an instability, because it is written in a Brans–Dicke frame. One can easily see that the kinetic terms for both moduli have the correct sign in the Einstein frame.

Introducing the new variables φ and ψ (Garriga *et al.*, 2001; Khoury *et al.*, 2001),

$$\varphi_+ = \varphi \cosh\psi \text{ and } \varphi_- = \varphi \sinh\psi, \tag{2.12}$$

the Einstein frame is given by $\hat{g}_{\mu\nu} = \varphi^2 \tilde{g}_{\mu\nu}$ In this frame, the action (2.10) takes the form

$$S_4[\varphi, \psi] = -m_{\mathrm{P}}^2 \int d^4x \sqrt{-\hat{g}} \left\{ \hat{\mathcal{R}} + \frac{2}{3}(\hat{\partial} \ln \varphi)^2 + \frac{4}{3}(\hat{\partial}\psi)^2 \right\}, \tag{2.13}$$

and now the kinetic terms are both positive definite.

2.3. Coupling of Moduli to Matter Fields

We note that the metric in the 4D Einstein frame $\hat{g}_{\mu\nu}$ does not correspond to the metric on either of the two branes, and so matter on the branes will not couple universally to it. Rather, 6D matter localized on the negative tension brane couples universally to g^-_{ab}, the metric induced by g_{ab} at $z = z_-$ In the 5D theory this corresponds to a universal coupling to $g^{(s)-}_{\mu\nu}$ (not $g^{(5)-}_{\mu\nu}$).

To work out the coupling of φ and ψ to the matter located on the negative tension brane, we note that this kind of matter couples universally to the induced metric on the brane. Therefore, the action for the 5D matter fields $\Psi^{(5)}(x^\mu, x^5)$ is of the form (see (2.1)):

$$S^{\mathrm{matt}} = \int d^5x \sqrt{g_-}\mathcal{L}^{(5)}\left(\Psi^{(5)}, g^-_{ab}\right). \tag{2.14}$$

Inserting the dimensional reduction ansatz (2.4), we obtain

$$\begin{aligned} S^{\mathrm{matt}} &= \int d^4x\, d\theta \sqrt{g_{(s)-}}\, e^{\sigma(z_-)}\mathcal{L}^{(5)}\left(\Psi^{(5)}, g^{(s)-}_{ab}\right) \\ &\quad \times \int d^4x\, d\theta \sqrt{g_{(s)-}}\, a_- \mathcal{L}^{(4)}\left(\Psi^{(4)}, g^{(s)-}_{\mu\nu}\right), \end{aligned} \tag{2.15}$$

where $g^{(s)-}_{\mu\nu} = a_-^2 \tilde{g}_{\mu\nu}$, $\Psi^{(4)}(x^\mu)$ are the S^1 zero modes of the matter fields and the coordinate volume factor $2\pi R$ has been absorbed in their normalization and couplings. Then, matter interacts only with the modulus a_- through

$$S^{\mathrm{mod-matt}} = -\int d^4x \sqrt{-g_{(s)-}}\,\mathcal{T}\,\frac{\delta a_-}{a_-}, \tag{2.16}$$

where $\mathcal{T} = T_{\mu\nu} g^{\mu\nu}_{(s)-} + a\mathcal{L}^{(4)}$, and $\mathcal{T}_{\mu\nu}$ is the 4D energy momentum tensor of the matter fields $\Psi^{(4)}$ computed with the metric $g^{(s)-}_{\mu\nu}$, $T_{\mu\nu} = -(2/\sqrt{g_{(s)-}})\delta S^{\mathrm{matt}}/\delta g^{\mu\nu}_{(s)-}$. We note that this coupling of the moduli to the lagrangian is entirely due to the circle being warped.[6]

Since a small perturbation $\delta\varphi_-$ around some v.e.v. φ_- can be expressed in terms of φ and ψ defined in (2.12) as $\delta\varphi_- = a^{3/2}\delta\varphi + \delta\psi$,[7] we can express the interaction with the canonical moduli (2.16) in the Einstein frame as

$$S^{\mathrm{mod-matt}} = -\frac{2}{3}\int d^4x \sqrt{-\hat{g}}\{\hat{\mathcal{T}}\delta\,\ln\varphi + a^{-3/2}\hat{\mathcal{T}}\delta\psi\}. \tag{2.17}$$

Here, $\hat{\mathcal{T}}$ is the same as $\mathcal{T}$ but computed with the 4D Einstein frame metric $\hat{g}_{\mu\nu}$, so that $\hat{\mathcal{T}} = (a_-/\varphi)^4\mathcal{T}$. Defining the canonical fields

$$\hat{\varphi} = \frac{2}{\sqrt{3}} m_{\mathrm{P}} \ln\varphi \quad \text{and} \quad \hat{\psi} = \frac{4}{\sqrt{3}} m_{\mathrm{P}}\psi,$$

we obtain the equations of motion for the moduli

$$\hat{\Box}\hat{\varphi} = \frac{2}{\sqrt{3}m_{\mathrm{P}}}\hat{\mathcal{T}} \quad \text{and} \quad \hat{\Box}\hat{\psi} = \frac{1}{\sqrt{6}m_{\mathrm{P}}} a^{-3/2}\hat{\mathcal{T}}, \tag{2.18}$$

so that $\hat{\varphi}$ couples to the matter on the negative tension brane with a strength $\sim 1/m_{\mathrm{P}}$ and $\hat{\psi}$, with a quite larger strength, $\sim a^{-3/2}/m_{\mathrm{P}}$.

2.4. Scales and Hierarchy

In this section we discuss the constraints for both the the dynamical moduli $R_\pm$ and the fixed scales, M and k, as well as the geometrical interpretation of the Planck/EW hierarchy.

On the one hand, the smallest physical length scale is given by the size of S^1 at the negative tension brane, $R_- = a_- R$. This cannot be smaller than the fundamental length of the theory, M^{-1}. Since the warp factor of the circle is the same as the Minkowski factor, the physical masses m_{S^1} of the first KK excitations along the circle are of order $1/R$, for matter at either z_+ or z_-. Collider experiments require $m_{S^1} \gtrsim TeV$.

[6] Actually, $\mathcal{T}$ coincides with the trace of the 5D energy momentum tensor.
[7] In the case of our interest, a $\ll 1$ in order to have a large hierarchy arising from the warp factors, as we explain in Section 2.4. Moreover, we can always take $\langle a_+ \rangle) = 1$. So, with a good accuracy, $\varphi \approx \varphi_+ \sim 1$ and $\psi \approx \varphi_- \ll 1$.

However, we advance here that the result for the potential can be expanded as a power series in $r_{\pm} \equiv R_{\pm}/\ell$, valid when both combinations are small. In this paper we focus on this regime, corresponding to the physical situation when the size of the internal manifold S^1 is *everywhere* smaller than the interbrane distance $\sim \ell$. Accordingly, this indicates that we have to assume that the separation between the fundamental cutoff M and the curvature scale $1/\ell$ of the background is at least of order a. Supersymmetry might be needed to stabilize this hierarchy of scales, so that in this model SUSY is not substituted by any other mechanism to stabilize Planck/EW hierarchy. However, SUSY is a common ingredient in theories coming from M theory, and the aim of this paper is to find out whether or not in such models the quantum effects can stabilize the moduli. So, we can consider a scenario such as a 6D SUSY theory with particles that get a mass of order the SUSY breaking scale $\sim 1/\ell$.

On the other hand, the mass m_{RS} of the first KK excitations along the orbifold is of order a/ℓ as in the RS model), and the bound from collider physics is $(1/\ell) \gtrsim a^{-1}$ *TeV*. This leads to consider the involved scales distributed as in Fig. 1.

Now we can obtain an expression for the EW/Planck hierarchy in this model. Taking into account the considered values for the scales and moduli, and from the relation between the 4D effective Planck mass and the higher-dimensional one Eq. (2.11)

$$m_{\mathrm{P}}^2 \sim a^{-2} M^2,$$

we already note a *large volume* factor a^{-2}.

Because of the warp factor in (2.2), particles living at z_-, with a physical mass $\sim 1/\ell$, have a redshifted mass given by $(a_-/a_+)(1/\ell) = a(1/\ell)$, as observed

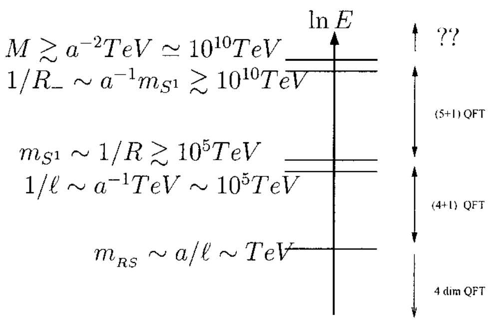

Fig. 1. The smallest scale m_{RS} is set to be $\sim$ TeV. By the condition that S^1 is everywhere smaller than the orbifold, $R_{\pm}/\ell \ll 1$, $R = R_+$ is forced to lie a hierarchical factor $1/a$ above. Since $R_-/R_+ = a$, R_- lies another factor $1/a$ above. And we take the simplest possibility when the fundamental cutoff is not far from this scale.

from the brane at z_+. Supposing that these are the particles in the Standard Model, the EW/Planck hierarchy is then given by

$$h^2 \equiv a^2 \frac{1/\ell^2}{m_{\rm P}^2} \sim \frac{a^2}{RM(\ell M)^3} \sim 10^{-32}. \tag{2.19}$$

Taking into account the optimal values for these scales, we obtain

$$h \sim a^3. \tag{2.20}$$

In contrast with the the RS model, where the hierarchy h coincides with the ratio of warp factors, a here there are two additional powers. Of these, one comes from the *large volume* factor noted earlier. The other one comes from considering the masses of the particles of order $1/\ell$ instead of M.

This shows how the problem of stabilization compatible with a large hierarchy works. Having introduced a certain hierarchy $\sim a$ between $1/\ell$ and M in (2.19), and the scale of the physical masses of particles $\sim 1/\ell$, we wonder whether or not the potential (3.17) can stabilize the moduli $R_\pm$ near the optimal values $R_+ \lesssim \ell$ and $R_- \gtrsim 1/M$, i.e., so that h^2 is enhanced by three more powers of the same hierarchy a. If this happens, there are three *input* powers of a in (2.19) and three more are generated, in this model. We address this question in Section 4.

3. POTENTIAL INDUCED BY MASSIVE BULK SCALAR FIELDS

In this section we compute the contribution to the one-loop effective potential due to a massive bulk scalar field χ, using dimensional regularization. Its action is given by

$$\begin{aligned} S_\chi = \frac{1}{2}\int d^{D+1}x\,\chi[-\Box - m^2]\chi - \frac{1}{2}\int d^D x\sqrt{g_+}m_+\chi^2 \\ -\frac{1}{2}\int d^D x\sqrt{g_-}m_-\chi^2. \end{aligned} \tag{3.1}$$

We shall dimensionally regularize the Minkowskian directions only so that the space-time topology is $M^{D-1} \times S^1 \times S^1/Z_2$ with $D-1 = 4-\epsilon$. The bulk equation of motion is

$$\left[\partial_z^2 - \frac{D-1}{z}\partial_z - \frac{(\ell m)^2}{z^2} + \Box_0 + R^{-2}\partial_0^2\right]\chi = 0, \tag{3.2}$$

with $\Box_0$ the 4D flat D'Alembertian. This equation admits a factorization in modes of the form

$$\chi_{n,m,k} = e^{ik_\mu x^\mu + im\theta/R}\, f_n(z), \tag{3.3}$$

with f_n solving

$$\left[\partial_z^2 - \frac{D-1}{z}\partial_z - \frac{(\ell m)^2}{z^2} + m_n^2\right] f_n(z) = 0, \tag{3.4}$$

where $-k^\mu k_\mu = m_n^2 + (m/R)^2$ are the physical masses. This implies

$$f_n(z) = z^{(D/2)}(A_n J_\nu(m_n z) + B_n Y_n(m_n z)), \tag{3.5}$$

where[8]

$$\nu = \sqrt{D^2/4 + m^2\ell^2}. \tag{3.6}$$

Imposing Dirichlet (appropriate for an odd Z_2 parity field) boundary conditions on both branes, we obtain the equation that defines implicitly the discrete spectrum of m_n,

$$F^{(D)}(\tilde{m}_n) = J_\nu(\tilde{m}_n a)Y_\nu(\tilde{m}_n) - J_\nu(\tilde{m}_n)Y_\nu(\tilde{m}_n a) = 0, \tag{3.7}$$

where we have defined

$$\tilde{m}_n = m_n z_-, \qquad a = \frac{z_+}{z_-} = \frac{R_-}{R_+}. \tag{3.8}$$

Even Z_2 parity fields obey boundary conditions of Neumann type $\partial_y - m_\pm/\ 2)\chi|_\pm = 0$, and the spectrum is determined by

$$F^{(N)}(\tilde{m}_n) = j_\nu^+(\tilde{m}_n a)y_\nu^-(\tilde{m}_n)j_\nu^-(\tilde{m}_n)y_\nu^+(\tilde{m}_n a) = 0, \tag{3.9}$$

where we defined

$$\begin{aligned} j_{\nu-1}^\pm(z) &\equiv J_{\nu-1}(z) + \frac{\alpha_\pm - \nu + D/2}{z} J_\nu(z), \\ y_{\nu-1}^\pm(z) &\equiv Y_{\nu-1}(z) + \frac{\alpha_\pm - \nu + D/2}{z} Y_\nu(z), \end{aligned} \tag{3.10}$$

where $\alpha_\pm = -m_\pm \ell/2$

We shall compute the effective potential in dimensional regularization. First, we shall work out the dimensionally regularized potential V^D, and then we need to subtract the *covariant* divergent piece V^{div}. This is of the form (Garriga *et al.*, 2001)

$$V^{\text{div}} = \frac{1}{(D-5)} \frac{1}{\mathcal{A}} a_{6/2}^D,$$

where $a_{6/2}^D$ is the relevant Seeley–DeWitt coefficient in six dimensions, we have regularized the dimensions orthogonal to the orbifold as $D = 5 - \epsilon$, and $\mathcal{A}$ is the

[8] A coupling to the curvature of the form $-\xi\mathcal{R}\chi^2$ can be automatically included by adding $-\xi D(D+1)/\ell^2$ to the bulk mass m and $\pm 4\xi D$ to the boundary masses $m_\pm$. For Dirichlet fields the latter are not relevant, but we keep them for generality.

4D comoving volume $\int d^{4-\epsilon}x$. Schematically, then we have

$$V = \lim_{D\to 5}[V^D - V^{\text{div}}]. \tag{3.11}$$

The dimensionally regularized potential V^D is given by

$$V^D \equiv \mu^\epsilon \sum_n \sum_m \frac{1}{2} \int \frac{d^{D-1}k}{(2\pi)^{D-1}} \ln\left[\frac{k^2 + m_n^2 + (m/R)^2}{\mu^2})\right]. \tag{3.12}$$

Performing the momentum integrals in (3.12),

$$V^D = -\frac{1}{2(4\pi)^2}\frac{(4\pi\mu^2R^2)^{s+2}}{R^4}\Gamma(s)\sum_{n,m}(m^2 + R^2m_n^2)^{-s}. \tag{3.13}$$

We can split the sum in three parts: the contributions coming from the orbifold zero mode (corresponding to $n = 0$), the circle zero mode (corresponding to $m = 0$), and the nonzero modes,

$$\sum_{n,m}(m^2 + R^2m_n^2)^{-s} = 2\sum_{m=1}^{\infty} m^{-2s} + \sum_{n=1}^{\infty}(2m_n)^{-2s} + 2 \times \sum_{n,m=1}^{\infty}(m^2 + R^2m_n^2)^{-s} \tag{3.14}$$

$$= 2\zeta_{\text{R}}(2s) + \left(\frac{R}{z_-}\right)^{-2s}\hat{\zeta}(2s) + 2\sum_{m=1}^{\infty} m^{-2s} \times \sum_{n=1}^{\infty}(1 + R^2m_n^2/m^2)^{-s}, \tag{3.15}$$

where ζ_{R} is the Riemann zeta function and the generalized zeta function $\hat{\zeta}$ has been evaluated in the literature (Flachi cand Toms, 2001; Garriga *et al.*, 2000, 2001; Goldberger and rothstein, 2000). Here we have assumed that the bulk field is even otherwise the orbifold zero mode would not be present. The problem now reduces to evaluating the third term of Eq. (3.14). Defining

$$x = \frac{R}{mz_-},$$

each term in the sum over m can be expressed as a contour integral

$$\sum_n(1 + x^2\hat{m}_n^2)^{-s} = \frac{1}{2\pi i}\int_{\mathcal{C}} dt(1 + x^2t^2)^{-s}\frac{F'(t)}{F(t)}$$

$$= \frac{sx^2}{\pi i}\int_{\mathcal{C}} t\, dt(1 + x^2t^2)^{-s-1}\ln F(t), \tag{3.16}$$

on the contour $\mathcal{C}$ of Fig. 2.

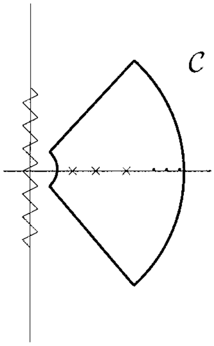

Fig. 2. The contour used to sum 3.16.

This can be analytically continued to the physical value of $s = -2$, and we do it in Appendices A and B for Dirichlet and Neumann boundary conditions. Here, we simply quote the final result for V, in terms of $r_\pm \equiv R_\pm/\ell$,

$$V(r_+, r_-) = \frac{1}{(4\pi)^2 R^4}[V_+(r_+) + V_-(r_-)v(r_+, r_-)], \tag{3.17}$$

with asymptotic expressions for

$$\begin{aligned} V_\pm(R_\pm) \sim{}& \pm a_{-1}\frac{1}{r_\pm} + a_0 \\ & \pm a_1^{(\pm)} r_\pm + a_2^{(\pm)} r_\pm^2 \pm a_3^{(\pm)} r_\pm^3 \\ & + a_4^{(\pm)} r_\pm^4 \ln r_\pm \pm a_5^{(\pm)} r_\pm^5 \ln(r_{\pm\mu_\pm}\ell) + \sum_{k=6}^{\infty} a_{\mathrm{i}}^{(\pm)} (\pm r_\pm)^k, \end{aligned} \tag{3.18}$$

where the coefficients $a_k^{(\pm)}$ are polynomials of degree k on the bulk mass m and the corresponding brane mass $m_\pm$. The first coefficient a_{-1} is a numerical constant and the sign of a_0 depends only on the type of boundary condition. We also introduced one renormalization constant for each brane $\mu_\pm$. We see from (2.1) that a finite renormalization of the brane tensions $\delta\sigma_\pm$ gives a term in the potential of the form

$$\frac{1}{\ell^4} R_\pm^5 \delta\sigma_\pm \tag{3.19}$$

Consequently, in the following we will include the In $\mu_\pm$ term inside $\delta\sigma_\pm$. We note that (3.18) are only asymptotic expansions and are valid only in the regime $r_\pm < 1$. The nonlocal part $\upsilon(r_+, r_-)$ is given in the Appendices. Here, we note that it is safely negligible in the limit of everywhere small internal space $R_\pm \ll \ell$, and large warping $a \ll 1$ (see Eqs. (A16) and (A19)).

4. STABILIZATION

We shall focuss the discussion on the stabilization of the moduli in the regime of large orbifold size $R_\pm/\ell \ll 1$ and large warping $a \ll 1$, because under this condition the nonlocal part of the potential $\upsilon(R_+, R_-)$ is negligible, and the potential separates as $V \sim (1/(4\pi)^2 R^4)[V_+(R_+) + V_-(R_-)]$. To further simplify the structure of V, we assume a matter content with equal number of bosons and fermions. This makes the two first terms in $V_\pm$ to vanish because neither a_0 nor a_{-1} depends on the mass of the field.

To stabilize the moduli $R_\pm$, we shall look for the values of $\delta\sigma_\pm$ (corresponding to a finite renormalization of the brane tensions) such that the conditions of extremum $\partial_{r\pm} V = 0$ together with having an effective 4D cosmological constant, $V|_{\min} \approx 10^{-122} m_{\mathrm{P}}^4$ can be satisfied. Since V_+ is essentially a polynomial in $r_+ = R_+/\ell$ with coefficients $\lesssim 1$, the extrema are typically at $R_+ \sim \ell$. In order for the result (3.18) to be convergent, we will assume that actually $R_+ \lesssim \ell$. We shall neglect the higher powers in the potential except for the brane tension renormalization terms. So the zero cosmological constant and the extremum condition for r_+ are

$$V_+ \approx a_1^+ r_+ + a_2^+ r_+^2 + \ldots a_5^+ r_+^5 \ \ln(\mu_+ \ell r_+) = 0$$
$$\partial_{r+} V \approx a_1^+ + 2a_2^+ r_+ + \ldots + 5a_5^+ r_+^4 [\ln(\mu_+ \ell r_+) + 1/5] = 0. \qquad (4.1)$$

We identify $\mathrm{In}(\mu + \ell)$ as part of the finite renormalization of the brane tensions $\delta\sigma_+/\ell^5$. With this, we find a solution of this equation for $r_+ \sim -4a_1^+/(3a_2^+)$[9], and

$$\ell^5 \delta\sigma_+ \sim \frac{a_1^+}{3a_5^+} \frac{1}{r_+^4}.$$

For the r_- modulus, we have

$$\partial_{r-} V \approx -a_1^- - 5a_5^- r_-^4 [\ln(\mu_- \ell r_-) - 1/5] = 0, \qquad (4.2)$$

and now we simply choose

$$\ell^5 \delta\sigma_- \sim -\frac{a_1^+}{5a_2^+} \frac{1}{r_-^4}$$

to get a v.e.v equal to r_-. Thus, the sizes of the renormalization of the brane tensions are

$$\delta\sigma_+ \sim 1/\ell^5,$$
$$\delta\sigma_- \sim a^{-4}/\ell^5. \qquad (4.3)$$

[9] We can easily check that for a Dirichlet field this ratio is $\simeq 0.6$, in agreement with the assumption made earlier. For the Neumann case, this ratio depends on the boundary and bulk masses, so that generically, it can be made small.

Comparing these to the bare values $|\sigma_\pm| \sim M^4/\ell \sim a^{-4}(1/\ell)^5$, we conclude that we have to do one fine-tuning. However, this corresponds to matching the observed value of the cosmological constant.

Next, we ask for the masses that this mechanism induces for the moduli $\hat{\varphi}$ and $\hat{\psi}$. A straightforward evaluation of $\partial^2_{\hat{\varphi}\pm} V|_{\min} = (\partial_{\hat{\varphi}\pm} r_\pm)^2 \partial^2_{r\pm} V_\pm|_{\min}$ gives[10]

$$m^2_{\hat{\varphi}} \sim a_2^+ h^{-2/3}(1/mm)^2 \sim KeV^2,$$
$$m^2_{\hat{\varphi}} \sim -a_1^- TeV^2. \tag{4.4}$$

The mass for the modulus $\hat{\varphi}$ generated by the effective potential is large enough not to violate Newton's law at short distances. Its small coupling to matter on the negative tension brane, suppressed by a planckian factor (see Eq. (2.18)), renders it unobservable in collider experiments. On the other hand, we obtain a mass for $\hat{\psi}$ of order of some *TeV*. From Eq. (2.18), this modulus couples to matter with a strength of order $1/10^8$ *TeV*, which is much larger than planckian but still leads to unobservable effects.

We conclude that, depending on the bulk field content, the effective potential can stabilize both moduli without fine-tuning, although the price to pay in this model is the introduction of a separation $a \sim 10^5$ between $1/\ell$ and M. It is shown in Flachi *et al.* (2003) that this feature improves with a larger number of flat extra dimensions. That is, in toroidal compactifications of a slice of higher-dimensional AdS the effective potential gives sizeable masses to the moduli, and the separation between M and $1/\ell$ decreases.

5. DISCUSSION AND CONCLUSIONS

We have presented the computation of the effective potential induced by massive bulk scalar fields nonminimally coupled to curvature in a slice of AdS_6 compactified on a circle S^1, for both Dirichlet and Neumann boundary conditions. This space-time is a generalization of the RS model with more extra dimensions, and we consider it as a first approximation to brane models arising in M theory with warped additional compact extra dimensions.

We have focussed on the situation when the size of the additional space S^1 is smaller than the orbifold size. We have shown the equivalence of the 6D (Einstein–Hilbert) theory compactified on the circle with the scalar–tensor 5D theory with an exponential potential (2.7), considered in Garriga *et al.* (2001). We followed the analysis presented there, on the basis of the *moduli approximation*, to obtain the effective 4D gravity with two scalar fields (the *moduli*) of Brans–Dicke type (2.13), describing the position of the branes on the orbifold (or equivalently, the

[10] Since $\zeta'(-3) > 0$, and the β_1 tabulated in the Appendices for scalar fields, the condition for negative a_1^- is that either we have fermion fields, or is the bulk field nonminimally coupled.

interbrane distance and the radius of S^1). As well, we have derived the couplings of each modulus to higher-dimensional matter located on the negative tension brane.

The computation of the effective potential is especially simple when the interbrane distance is larger than S^1. We have proposed the setup of scales that corresponds to this regime, and shown that the Planck/EW hierarchy is generated by a combination of a large volume effect (Antoniadis *et al.*, 1998; Arkani-Hamed *et al.*, 1998, 1999) and a redshift induced by the warp factor (Randall and Sundrum, 1999). In this model, we have to introduce some hierarchical separation $a \sim 10^5$ between the fundamental cutoff and the curvature scale of the background. However, the separation between the EW scale and the Planck mass m_P is given by a^3.

The resulting Casimir energy (3.17, 3.18) can stabilize the moduli at locations compatible with the observed Planck/EW hierarchy $\sim 10^{16}$ without fine-tuning (except for a tuning of the positive tension σ_+, needed to match the cosmological constant), generating large enough masses to give unobservable effects. However, the particular choice of the parameters of the model (M, $1/\ell$, and the mass of the particles) can be thought of a tuning. Anyhow, as shown in Flachi *et al.* (2003) these features improve when we consider a toroidal compactification of a slice of higher-dimensional *AdS* spaces.

Garriga *et al.* (2001) raised the question that the path integral measure of a bulk scalar field in the effective 5D theory (2.7) quantized on the warped vacuum configuration (2.8) is ambiguously defined. The nontrivial profile of the scalar ϕ permits to define many different conformal frames, all of them equivalent at the classical level. However, the path integral measure can be defined covariantly with respect to any of them. It turns out that the term proportional to

$$\frac{\ln z_+}{z_+^4} + \frac{\ln z_-}{z_-^4}$$

in the potential depends on this choice. Several arguments can be given in favor of possible 'preferred' frames. For instance, with a measure covariant with respect to the 5D Einstein frame metric, this term is present. But if one chooses covariance with respect to the 6D Einstein frame metric, there is no such term. However, in the model presented here, there is no ambiguity in the choice of the measure since the 6D theory is Einstein–Hilbert gravity, and there is no scalar. In the computation presented here, the choice of the measure shows up when we subtract the divergences in the potential Eq. (A20), covariant precisely with respect to the 6D Einstein frame metric. As a result when we take into account both the 5D modes (the S^1 KK zero mode) together with the 6D ones (the KK modes excited along S^1 as well), we have found that there is a remaining contribution of this form (see Eqs. (3.17) and (3.18). Anyhow, it should be noted that these Coleman–Weinberg-like terms do not play a very relevant role in stabilizing the moduli.

ACKNOWLEDGMENTS

I thank Enric Verdaguer and Edgar Gunzig for their kind invitation to the "Peyresq-7 Meeting," and Jaume Garriga, Takahiro Tanaka, and Antonino Flachi for useful discussions. I acknowledge support by CICYT under Grant No. AEN99-0766.

6. APPENDIX A: DIRICHLET BOUNDARY CONDITIONS

In this Appendix we evaluate the sum (3.16) and show the main steps leading to Eqs. (3.17) and (3.18) for the effective potential. For the sake of clarity we consider the case of Neumann boundary conditions separately. The integral

$$I = \int_{\mathcal{C}} t\, dt (1 + x^2 t^2)^{-s-1} \ln F^{(D)}(t), \tag{A1}$$

appearing in (3.16) is defined for $\Re s > 1/2$ but it can be continued to the left in the plane s, and $x = R/mz_- = r_-/m$. Using the asymptotic expansions of the Bessel functions for large argument

$$K_\nu(\rho) = \sqrt{\frac{\pi}{2\rho}} e^{-\rho} C_\nu(\rho),$$

$$I_\nu(\rho) = \frac{e^\rho}{\sqrt{2\pi\rho}} C_\nu(-\rho) + o(e^{-\rho}), \tag{A2}$$

where

$$C_\nu(\rho) \sim \sum_{k=1}^{\infty} \frac{\Gamma(\nu + k + 1/2)}{k!\Gamma(\nu - k + 1/2)} (2\rho)^{-k},$$

we can work out the leading term in the asymptotic expansion for $F^{(D)}(t)$,

$$F^{(D)}(t) \sim \frac{\sigma \mathrm{i}\, e^{-\sigma \mathrm{i}(1-a)t}}{\pi \sqrt{at}} (1 + o(1/t)) \quad \text{at} \mathcal{C}^\sigma, \tag{A3}$$

with $\sigma = \pm$ and $\mathcal{C}^\pm$ the upper and lower halves of the contour in Fig. A1. Multiplying and dividing by the asymptotic form of $F^{(D)}$ (A3) inside the logarithm, we can split I into

$$I = I_1 + I_2 \equiv \sum_{\sigma=\pm} \left\{ \int_{\mathcal{C}_2^\sigma} t\, dt (1 + x^2 t^2)^{-s-1} \ln\left(-\sigma i t \pi \sqrt{a}\, e^{\sigma i(1-a)t} F^{(D)}(t) \right) \right. \tag{A4}$$

$$\left. - \int_{\mathcal{C}_2^\sigma} t\, dt (1 + x^2 t^2)^{-s-1} \ln\left(-\sigma \mathrm{i} t \pi \sqrt{a}\, e^{\sigma \mathrm{i}(1-a)t} \right) \right\}. \tag{A5}$$

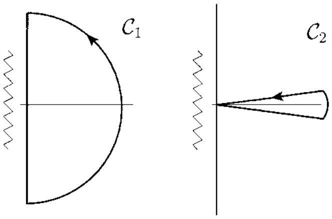

Fig. A1. We can deform the contour $\mathcal{C}$ in figure as $\mathcal{C}_1$ and $\mathcal{C}_2$ in order to commute I_1 and I_2.

Here, since the poles of the integrand lie on the real axis, we have deformed the integration contour to $\mathcal{C}_{1,2}$ for each piece (see Fig. A1).

The integral I_2 can be readily evaluated,

$$\begin{aligned} I_2 &= \sum_\sigma \sigma \int_0^\infty \rho\, d\rho (1+x^2\rho^2)^{-1-s} \ln\left(-\sigma \mathrm{i}\pi\sqrt{a}\rho\, e^{\sigma \mathrm{i}(1-a)\rho}\right) \\ &= \int_0^\infty \rho\, d\rho (1+x^2\rho^2)^{-1-s}[-\mathrm{i}\pi + 2(1-a)\rho \mathrm{i}] \\ &= -\frac{\pi}{2}\frac{1}{sx^2}\mathrm{i} + \frac{\sqrt{\pi}}{2}\frac{\Gamma(s-1/2)}{\Gamma(1+s)}\frac{1-a}{x^3}\mathrm{i}. \end{aligned} \tag{A6}$$

This integral converges for $\Re s > 1/2$, and the result suggests itself as the analytic continuation to the left of the complex plane of s.

We are ready to work out the contribution from this part of I to the effective potential, for the Dirichlet boundary conditions. From (3.14) and (3.13), and performing the sum over m, this is given by

$$\begin{aligned} &-\frac{1}{(4\pi)^2}\frac{(4\pi\mu^2R^2)^{s+2}}{R^4}\Gamma(s)\left(-\frac{1}{2}\zeta_R(2s) + \frac{\sqrt{\pi}\,\Gamma(s-1/2)}{\Gamma(s)}\zeta_R(2s-1)\right. \\ &\left.\times (1-a)\frac{\ell}{R_-}\right), \end{aligned} \tag{A7}$$

which is finite at $s \to -2$. Since an odd parity bulk field has no orbifold zero mode, these are all the contributions of this form that we have for the Dirichlet field (see Eqs. (3.13) and (3.14)),

$$\frac{1}{(4\pi)^2}\frac{1}{R_4}\left[\frac{1}{2}\zeta_R'(-4) - \frac{1}{945}\frac{\Delta z}{R}\right], \tag{A8}$$

where $\Delta z \equiv z_- - z_+$.

To compute I_1, we separate the integration along the imaginary axis from 0 to the branch point $\pm \mathrm{i}/x$ and from the branch point to $\pm\infty$,

$$I_1 = \int_0^{1/x} \rho\, d\rho (1-x^2\rho^2)^{-s-1} \ln\left[\pi\sqrt{a}\rho\, e^{-(1-a)\rho}F_D(\mathrm{i}\rho)\right]$$

$$
\begin{aligned}
&+e^{-\mathrm{i}(s+1)\pi}\int_{1/x}^{\infty}\rho\, d\rho (x^2\rho^2-1)^{-s-1}\,\ln\left[\pi\sqrt{a}\rho\, e^{-(1-a)\rho}F_D(\mathrm{i}\rho)\right]\\
&-\int_0^{1/x}\rho\, d\rho(1-x^2\rho^2)^{-s-1}\,\ln\left[\pi\sqrt{a}\rho\, e^{-(1-a)\rho}F_D(-\mathrm{i}\rho)\right]\\
&-e^{-\mathrm{i}(s+1)\pi}\int_{1/x}^{\infty}\rho\, d\rho (x^2\rho^2-1)^{-s-1}\,\ln\left[\pi\sqrt{a}\rho\, e^{-(1-a)\rho}F_D(-\mathrm{i}\rho)\right]\\
&=-2\mathrm{i}\sin[(s+1)\pi]\int_{1/x}^{\infty}\rho\, d\rho (x^2\rho^2-1)^{-s-1}\,\ln\left(\pi\sqrt{a}\rho\, e^{-(1-a)\rho}F_D(\mathrm{i}\rho)\right),
\end{aligned}
$$

valid for $-1/2 < \Re s < 0$. Here we used that $F^D(-z) = F^D(z)$. The analytic continuation of F^D to the imaginary axis gives

$$
F_D(\mathrm{i}\rho) = \frac{2}{\pi}[I_\nu(\rho)K_\nu(a\rho) - I_\nu(a\rho)K_\nu(\rho)]. \tag{A9}
$$

We can rewrite I_1 as

$$
I_1 = \frac{2\mathrm{i}}{x^2}\sin(\pi s)\{\mathcal{I}_K(a,x) + \mathcal{I}_I(a) + \mathcal{V}_{(D)}(a,x)\}, \tag{A10}
$$

with

$$
\begin{aligned}
\mathcal{I}_K(a,x) &\equiv \int_1^\infty y\, dy(y^2-1)^{-s-1}\,\ln\left(\sqrt{\frac{2ay}{\pi x}}\, e^{ay/x}K_\nu(ay/x)\right),\\
\mathcal{I}_I(x) &\equiv \int_1^\infty y\, dy(y^2-1)^{-s-1}\,\ln\left(\sqrt{2\pi y/x}\, e^{-y/x}I_\nu(y/x)\right),\\
\mathcal{V}_{(D)}(a,x) &\equiv \int_1^\infty y\, dy(y^2-1)^{-s-1}\,\ln\left(1-\frac{I_\nu(ay/x)K_\nu(y/x)}{I_\nu(y/x)K_\nu(ay/x)}\right),
\end{aligned} \tag{A11}
$$

and we note that the two first terms converge for $-1/2 < \Re s < 0$, whereas the last converges for $\Re s < 0$.

Using the asymptotic expansions of K_ν and I_ν (A2), we can analytically continue this integral to the relevant point $s = -2$ and, as we will see, we will obtain an asymptotic expansion in powers of $R_\pm/\ell < 1$ for $\mathcal{I}_{I,K}$. Defining the coefficients β_k as

$$
\ln \mathcal{C}_\nu(\rho) \sim \sum_{k=1}^{\infty}\frac{\beta_k}{\rho^k},
$$

we expand the integrand for small x, and performing the y integrals

$$
\int_1^\infty dy(y^2-1)^{-1-s}y^{1-k} = \frac{\Gamma(-s)\Gamma(s+k/2)}{2\Gamma(k/2)}, \tag{A12}
$$

we obtain

$$\mathcal{I}_K(a,x) \sim \frac{\Gamma(-s)}{2} \sum_{k=1}^{\infty} \frac{\Gamma(s+k/2)}{\Gamma(k/2)} \frac{\beta_k x^k}{a^k}. \tag{A13}$$

Doing the same for $\mathcal{I}_I(x)$, we have

$$\mathcal{I}_I(x) \sim \frac{\Gamma(-s)}{2} \sum_{k=1}^{\infty} \frac{\Gamma(s+k/2)}{\Gamma(k/2)} \beta_k x^k + o(e^{-2/x}). \tag{A14}$$

Now we turn to $\mathcal{V}_{(D)}(a, xs)$, which is already finite for $s=-2$. Keeping the first term in the asymptotic expansion of the Bessel functions (A2), we obtain

$$\begin{aligned} \mathcal{V}_{(D)}(a,x) &\sim \int_1^{\infty} dy (y^3 - y) \ln\left(1 - e^{-2(1-a)y/x}\right) \\ &= -\frac{1}{8} \frac{x^2}{(1-a)^4} \left\{ 4(1-a)^2 \operatorname{Li}_3\left(e^{-2(1-a)/x}\right) \right. \\ &\quad \left. + 6(1-a)x \operatorname{Li}_4\left(e^{-2(1-a)/x}\right) + 3x^2 \operatorname{Li}_5\left(e^{-2(1-a)/x}\right) \right\} \end{aligned} \tag{A15}$$

Since the argument of the polylogarithms is very small in the region we are considering, we can approximate $\operatorname{Li}_k(z) \approx z$. Recalling that $x = r_-/m$ and summing over m, to leading order we find

$$\sum_{m=1}^{\infty} m^4 \mathcal{V}_{(\mathcal{D})}(a, r_-/m) \sim -\frac{1}{2} \frac{r_-^2}{(1-a)^2} e^{-2(1-a)/r_-}. \tag{A16}$$

Thus, this contribution is safely negligible in the limit of small internal space size $R_- \ll \ell$.

Ignoring it, we can sum over m the contribution coming from I_1, using Eqs. (A10), (A11), (A13), and (A14). Since this is a power series in m, the sum will give rise to Riemann zeta functions. Recalling that $x = r_-/m$, we find

$$\begin{aligned} \Gamma(s) \sum_{m=1}^{\infty} m^{-2s} \frac{s x^2}{\pi \mathrm{i}} I_1 &= \frac{s}{\pi} \sin(\pi s) \Gamma(s) \Gamma(-s) \sum_{k=1}^{\infty} \frac{\Gamma(s+k/2)}{\Gamma(k/2)} \\ &\times \zeta_R(2s+k) \beta_k [r_+^k + (-r_-)^k] + \mathcal{O}\left(\frac{r_-^2}{(1-a)^2} e^{-2(1-a)/r_-} \right) \end{aligned} \tag{A17}$$

Since we are interested in this expression near $s=-2$, we see that the divergent parts come from the Γ functions for $k = 2, 4$. However, since $\zeta_R(z)$ has a zero at $z=-2$, the pole coming from $k=2$ is cancelled. For $k=4$, instead, we have a pole because $\zeta_R(0)$ is finite and nonzero. There is another pole in s coming from the divergence in the Riemann zeta function $\zeta_R(z)$ at $z=1$, that is, for $k=5$.

To the contribution from the modes excited along the S^1, given by (A8) together with (A17), we have to add the contribution from the circle zero mode (see Eq. (3.14)). This coincides with the potential in the RS model and has been computed in the literature (Flachi and Toms, 2001; Garriga *et al.*, 2000, 2001; Goldberger and Rothstein, 2000; Toms, 2000),

$$\frac{1}{(4\pi)^2}\left[\left\{\left(\frac{1}{\epsilon}+\frac{\ln 2\pi}{2}-\frac{\gamma}{2}+\frac{1}{2}\ln(4\pi\mu^2\ell^2)\right)\beta_4+\beta_4'\right\}\left(\frac{1}{z_-^4}+\frac{1}{z_+^4}\right)\right.$$
$$\left.+\beta_4\left(\frac{1}{z_-^4}\ln\left(\frac{z_-}{\ell}\right)+\frac{1}{z_+^4}\ln\left(\frac{z_+}{\ell}\right)\right)+\frac{\mathcal{I}_K^0}{z_+^4}+\frac{\mathcal{I}_I^0}{z_-^4}+\frac{\mathcal{V}^0_{(D)}(a)}{z_-^4}\right]+\mathcal{O}(\epsilon), \tag{A18}$$

where $\mathcal{I}_I^0$ and $\mathcal{I}_K^0$ are unimportant constants that can be computed as in Garriga *et al.* (2000, 2001), and

$$\mathcal{V}^0_{(D)}(a)=\int_0^\infty d\rho\,\rho^3\ln\left(1-\frac{I_\nu(a\rho)}{I_\nu(\rho)}\frac{K_\nu(\rho)}{K_\nu(a\rho)}\right). \tag{A19}$$

This function behaves as $a^{2\nu}$ for $a\ll 1$, so it gives rise to a negligible contribution to the potential.

We see that the divergence in the contribution due to the circle zero mode Eq. (A18) is exactly cancelled by the pole corresponding to $k=4$ in Eq. (A17). So, we are left with the (higher-dimensional) divergence corresponding to $k=5$ only. This divergence is removed subtracting the appropriate combination of geometrical invariant in the $(D-1)+1+1$ regularized space-time, given by the Seeley–DeWitt coefficient $a_{6/2}$. Since our space is maximally symmetric, all the geometric invariants are proportional to some power of the curvature radius and are equivalent to brane tension terms. So, the computation of $a_{6/2}$ is trivial up to a global numerical factor. On the other hand, this can be fixed imposing that the pole term is exactly cancelled. With this, we obtain

$$V^{\mathrm{div}}=\frac{1}{\epsilon}\frac{1}{\mathcal{A}}a^D_{6/2}\propto\frac{1}{\epsilon}\frac{1}{\ell^5}\left[\left(\frac{\ell}{z_+}\right)^{5-\epsilon}-\left(\frac{\ell}{z_-}\right)^{5-\epsilon}\right]R=\frac{1}{\epsilon}\left[\frac{1}{z_+^5}-\frac{1}{z_-^5}\right]R$$
$$+\left[\frac{\ln(z_+/\ell)}{z_+^5}-\frac{\ln(z_-/\ell)}{z_-^5}\right]R+\text{finite} \tag{A20}$$

where $\mathcal{A}=\int d^4x$ is the comoving 4D volume. Comparing with (A17), we see that the global constant factor must be $4\beta_5/3$. The first β coefficients are listed below.

Finally, adding up (A8), (A17), (A18), and (A20), we find

$$V(R_\pm)\sim\frac{1}{(4\pi)^2R^4}\left[\sum_{k=-1}^{\infty}a_k\left\{r_+^k+(-r_-)^k\right\}+a_4\left\{r_+^4\ln r_++r_-^4\ln r_-\right\}\right.$$
$$+a_5\left\{r_+^5\ln(\mu\ell r_-)-r_-^5\ln(\mu\ell r_-)\right\}+\mathcal{I}_K^0r_+^4$$

$$
+\left(\mathcal{I}_I^0+\mathcal{V}^0_{(D)}(a)\right)r_-^4+\mathcal{O}(e^{-2(1-a)/r_-})\Bigg], \tag{A21}
$$

where we have redefined the renormalization constant μ again, and the coefficients a_k are

$$
\begin{aligned}
a_{-1} &= \frac{1}{945},\\
a_0 &= \frac{1}{2}\zeta_R'(-4),\\
a_k &= \frac{4\beta_k}{(k-4)(k-2)}\zeta_R(k-4), \quad \text{for} \quad k=1,3,6,7,8\ldots.\\
a_2 &= -2\zeta_R'(-2)\beta_2 \quad \text{for} \quad k=2,\\
a_4 &= \beta_4,\\
a_5 &= \frac{4}{3}\beta_5.
\end{aligned} \tag{A22}
$$

For Dirichlet boundary conditions, the values of the first β coefficients, which depend only on the bulk mass of the field m, are

$$
\begin{aligned}
\beta_1 &= ((m\ell)^2+6)/2,\\
\beta_2 &= (-(m\ell)^2-6)/4,\\
\beta_3 &= -(m\ell)^2((m\ell)^2+6)/24,\\
\beta_4 &= ((m\ell)^4+9(m\ell)^2+18)/8,\\
\beta_5 &= ((m\ell)^6-10(m\ell)^4-168(m\ell)^2-432)/80.
\end{aligned} \tag{A23}
$$

From this, it is clear that the effective potential induced by bulk fields can be cast as (3.17) and (3.18), understanding that for Dirichlet boundary conditions the coefficients $a_j^{\pm}=a_j$.

7. APPENDIX B: NEUMANN BOUNDARY CONDITIONS

For the Neumann boundary conditions, we first realize that $F^{(N)}(t)\sim t^{-2}$ for $t\to 0$. Thus, if we use the same integral representation on the same contour $\mathcal{C}_1$ as in the previous case, we obtain an unwanted contribution from this point (a ‘zero mode’ contribution that has already been taken into account in Eq. (3.14)). However, we can get rid of this contribution, considering $t^2F^{(N)}(t)$ instead of $F^{(N)}(t)$. Obviously, this does not affect the result since t^2 has no zero inside $\mathcal{C}$. Moreover, it allows us to deform this contour into $\mathcal{C}_1$ without crossing any singularity. So in this case the appropriate integral representation is[11]

$$\sum_{n=1}^{\infty}(1+x^2\hat{m}_n^2)^{-s} = \frac{1}{2\pi i}\int_C dt(1+x^2t^2)^{-s}\,\frac{d}{dt}\,\ln\left(t^2F^{(N)}(t)\right)$$
$$= \frac{sx^2}{\pi i}\int_C t\,dt(1+x^2t^2)^{-s-1}\,\ln\left(t^2F^{(N)}(t)\right). \qquad \text{(B1)}$$

Multiplying and dividing inside the logarithm by the leading asymptotic behavior of $t^2F^{(N)}$, except for a minus sign (which is the phase that t^2 acquires when we integrate along $\mathcal{C}_1$, now we split I into

$$I = I_1 + I_2 \equiv \sum_{\sigma=\pm}\left\{\int_{C_1^\sigma} t\,dt(1+x^2t^2)^{-s-1}\,\ln\left(\sigma \mathrm{i}\pi\sqrt{at}\,e^{\sigma \mathrm{i}(1-a)t}F^{(N)}(t)\right)\right. \qquad \text{(B2)}$$
$$\left. - \int_{C_2^\sigma} t\,dt(1+x^2t^2)^{-s-1}\,\ln(\sigma \mathrm{i}\pi\sqrt{a}\,e^{\sigma \mathrm{i}(1-a)t}/t)\right\}. \qquad \text{(B3)}$$

The integral I_2 can be readily evaluated,

$$I_2 = \sum_\sigma \sigma\int_0^\infty \rho\,d\rho(1+x^2\rho^2)^{-1-s}\,\ln(\sigma \mathrm{i}\pi\sqrt{a}\,e^{\sigma \mathrm{i}(1-a)\rho}/\rho)$$
$$= \int_0^\infty \rho\,d\rho(1+x^2\rho^2)^{-1-s}[\mathrm{i}\pi + 2(1-a)\rho \mathrm{i}]$$
$$= \frac{\pi}{2}\frac{1}{sx^2}\mathrm{i} + \frac{\sqrt{\pi}}{2}\frac{\Gamma(s-1/2)}{\Gamma(1+s)}\frac{1-a}{x^3}\mathrm{i}. \qquad \text{(B4)}$$

As before, this integral converges for $\mathcal{R} > 1/2$, and we continue the result to the left of the complex plane. We note that the only difference with respect to the Dirichlet case is the sign of the first term. Taking into account that in the Neumann case we also have the contribution coming from the orbifold zero mode in (3.14), we get

$$\frac{1}{(4\pi)^2}\frac{1}{R^4}\left[-\frac{3}{2}\zeta_R^1(-4) - \frac{1}{945}\frac{\Delta z}{R}\right]. \qquad \text{(B5)}$$

To compute I_1, we separate the integration along the imaginary axis from 0 to the branch point $\pm i/x$ and from the branch point to $\pm\infty$,

$$I_1 = -2\mathrm{i}\,sin[(s+1)\pi]\int_{1/x}^{\infty}\rho\,d\rho(x^2\rho^2-1)^{-s-1}\,\ln\left(\pi\sqrt{a}\rho\,e^{-(1-a)\rho}F^{(N)}(\mathrm{i})\rho)\right), \qquad \text{(B6)}$$

[11] In the Dirichlet case $F^{(D)} \sim$ constant for $t \to 0$, so we do not need any power of t to smooth it.

valid for $-1/2 < \mathcal{R}s < 0$. Here we used that $F^N(-z) = F^N(z)$. The analytic continuation of F^N to the imaginary axis gives

$$F^{(N)}(\mathrm{i}\rho) = \frac{2}{\pi}\left[i^+_{\nu-1}(\rho)\kappa^-_{\nu-1}(a\rho) - i^-_{\nu-1}(a\rho)\kappa^+_{\nu-1}(\rho)\right], \tag{B7}$$

where now

$$\begin{aligned} i^\pm_{\nu-1}(\rho) &\equiv I_{\nu-1}(\rho) + \frac{\alpha^\pm - \nu + D/2}{\rho} I_\nu(\rho), \\ \kappa^\pm_{\nu-1}(\rho) &\equiv K_{\nu-1}(\rho)\frac{\alpha^\pm - \nu + D/2}{\rho} K_\nu(\rho). \end{aligned} \tag{B8}$$

We can rewrite I_1 as

$$I_1 = \frac{2\mathrm{i}}{x^2}\sin(\pi s)\left\{\mathcal{I}_k(a,x) + \mathcal{I}_i(a) + \mathcal{V}^{(N)}(a,x)\right\}, \tag{B9}$$

where

$$\begin{aligned} \mathcal{I}_k(a,x) &\equiv \int_1^\infty y\,dy(y^2-1)^{-s-1}\,\ln\left(\sqrt{\frac{2ay}{\pi x}}\,e^{ay/x}k^-_{\nu-1}(ay/x)\right), \\ \mathcal{I}_i(x) &\equiv \int_1^\infty y\,dy(y^2-1)^{-s-1}\,\ln\left(\sqrt{2\pi y/x}\,e^{-y/x}i^+_{\nu-1}(y/x)\right), \\ \mathcal{V}^{(N)}(a,x) &\equiv \int_1^\infty y\,dy(y^2-1)^{-s-1}\,\ln\left(1 - \frac{i^-_{\nu-1}(ay/x)\kappa^+_{\nu-1}(y/x)}{i^+_{\nu-1}(y/x)\kappa^-_{\nu-1}(ay/x)}\right). \end{aligned} \tag{B10}$$

As in the previous case, the last term is finite for $\mathcal{R}s < 0$ behaves, to leading order, like

$$\mathcal{V}_{(\mathcal{N})}(a,x) \sim -\frac{1}{2}\frac{r_-^2}{(1-a)^2}\,e^{-2(1-a)/r_-}. \tag{B11}$$

From (A2), we can obtain the asymptotic expansions for $i_{\nu-1}$ and $\kappa_{\nu-1}$,

$$\begin{aligned} \kappa^\pm_{\nu-1}(\rho) &= \sqrt{\frac{\pi}{2\rho}}\,e^{-\rho}C^\pm_{\nu-1}(\rho), \\ i^\pm_{\nu-1}(\rho) &= \frac{e^\rho}{\sqrt{2\pi\rho}}C^\pm_{\nu-1} + o(e^{-\rho}), \end{aligned} \tag{B12}$$

with

$$c^\pm_{\nu-1}(\rho) = C_{\nu-1}(\rho) + \frac{\alpha^\pm - \nu + D/2}{\rho}C_\nu(\rho).$$

Defining the coefficients $\beta_k^{\pm}$ through

$$\ln\, c^{\pm}_{\nu-1} \sim \sum_{k=1}^{\infty} \frac{\beta_k^{\pm}}{\rho^k},$$

the expression analogous to (A17) that we obtain is

$$\Gamma(s)\sum_{m=1}^{\infty} m^{-2s}\frac{sx^2}{\pi i}I_1 = \frac{s}{\pi}\ \sin(\pi s)\Gamma(s)\Gamma(-s)\sum_{k=1}^{\infty}\frac{\Gamma(s+k/2)}{\Gamma(k/2)}\zeta_R(2s+k)$$
$$\times\left[\beta_k^+ r_+^k + \beta_k^-(-r-)^k\right] + \mathcal{O}\left(\frac{r^2-}{(1-a)^2}e^{-2(1-a)r_-}\right). \tag{B13}$$

Subtracting the divergences as in the previous case, we obtain for Neumann boundary conditions a potential of the form

$$V(r_\pm) \sim \frac{1}{(4\pi)^2R^4}\left[\sum_{k=-1}^{\infty}\{a_k^+r_+^k + (-1)^k a_k^- r_-^k\}\right.$$
$$+\{a_4^+r_+^4\ \ln r_+ + a_4^- r_-^4\ \ln r_-\} + \{a_5^+r_+^5\ \ln(\mu\ell r_-) - a_5^- r_-^5\ \ln(\mu\ell r_-)\}$$
$$\left. + \mathcal{I}_k^o r_+^4 + \left(\mathcal{I}_i^o + \mathcal{V}^o_{(N)}(a)\right)r_-^4 + \mathcal{O}\left(e^{-2(1-a)/r-}\right)\right], \tag{B14}$$

where

$$\mathcal{V}^0_{(N)}(a) = \int_0^{\infty} d\rho\ \rho^3\ \ln\left(1 - \frac{i^-_{\nu-1}(a\rho)}{i^+_{\nu-1}(\rho)}\frac{\kappa^+_{\nu-1}(\rho)}{\kappa^-_{\nu-1}(a\rho)}\right), \tag{B15}$$

$\mathcal{I}_i^0$ and $\mathcal{I}_k^0$ are unimportant constants defined as in Garriga *et. al.* (2000, 2001), and the coefficients $a_k^{\pm}$ are

$$a^{\pm}_{-1} = \frac{1}{945}$$
$$a_0^{\pm} = -\frac{3}{2}\zeta_R'(-4)$$
$$a_k^{\pm} = \frac{4\beta_k^{\pm}}{(k-4)(k-2)}\zeta_R(k-4)\ \ \text{for}\ \ k = 1, 3, 6, 7, 8\ldots$$
$$a_2^{\pm} = -2\zeta_R^1(-2)\beta_2^{\pm}\ \ \text{for}\ \ k=2,$$
$$a_4^{\pm} = \beta_4^{\pm}$$
$$a_5^{\pm} = \frac{4}{3}\beta_5^{\pm}. \tag{B16}$$

For Neumann boundary conditions, the values of the first β coefficients, depending on the bulk m and brane $m_\pm$ masses of the field,

$$
\begin{aligned}
\beta_1^\pm &= (2 + m_\pm\ell + (m\ell)^2)/2 \\
\beta_2^\pm &= (-4 + 8m_\pm\ell - (m_\ell)^2 + 2(m\ell)^2)/8 \\
\beta_3^\pm &= (8 + 12m_\pm\ell - 12(m_\pm\ell)^2 + (m_\pm\ell)^3 + 6(m\ell)^2 - 6m_\pm\ell(ml)^2 - (m\ell)^4)/24 \\
\beta_4^\pm &= (-16 - 32m_\pm\ell - 48(m_\pm\ell)^2 + 16(m_\pm\ell)^3 - (m_\pm\ell)^4 - 8(m\ell)^2 \\
&\quad -48m_\pm\ell(m\ell)^2 + 8(m_\pm\ell)^2(m\ell)^2_- - 8(m\ell)^4)/64 \\
\beta_5^\pm &= (32 + 80m_\pm\ell - 40(m_\pm\ell)^2 + 100(m_\pm\ell)^3 - 20(m_\pm\ell)^4 + (m_\pm\ell)^5 \\
&\quad -16(m\ell)^2 - 20m_\pm\ell(m\ell)^2 + 100(m_\pm\ell)^2(m\ell)^2 - 10(m_\pm\ell)^3(m\ell)^2 \\
&\quad -20(m\ell)^4 + 30m_\pm\ell(m\ell)^4 + 2(m\ell)^6)/160.
\end{aligned}
$$

From these results it is straightforward to show that the effective potential induced by bulk fields can be generally written in the form of Eqs. (3.17), and (3.18).

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